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BRIEF COMMUNICATIONS

PARTICLE VELOCITIES IN VERTICAL PNEUMATIC CONVEYING SYSTEMS

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The calculation of pressure gradients in solid particle conveying systems requires a knowledge of particle velocities.

In the following discussion it is shown that, in the case of large particles in vertical pipes, a general equation for particle velocities can be written in terms of the known particle and system properties and the empirical coefficients, C_D, C_{Dt} and λ_s, which relate the drag force and the solids friction force to the flow parameters. It is further shown that, for a given rate of solids flow, the excess pressure gradient due to the presence of the solids passes through a minimum as the airflow is changed and that at this minimum the particle velocity is independent of the solids friction factor λ_s. Moreover, if the multi-particle drag coefficient C_D is approximately equal to the single particle drag coefficient C_{Dt}, then at the minimum, the relative velocity between the air and the solids is approximately 41% greater than the single particle terminal velocity in air.

The flow of dilute suspensions of solid particles in air has been studied extensively and satisfactory explanations of the effect of a change in the airflow on the total pressure gradient and the stability of the suspension have been given (Zenz & Othmer 1960; Wen & Galli 1971; Kunii & Levenspiel 1969).

In these texts the phenomenon of saltation in horizontal flow is described together with the sudden increase in the system pressure gradient which occurs when, at constant solids flow, the airflow falls below the saltation value. The equivalent choking velocity in vertical flow has also been described and the more gradual increase in pressure gradient as the airflow reduces beyond a pressure gradient minimum is recognised as being due to a fall in particle "friction" and to a rise in static head as the mixture density increases at lower air and particle velocities.

A knowledge of particle velocities is necessary for predicting pressure drops accurately in such systems but no generally accepted analytical method exists which enables these velocities to be calculated. Some investigators have concluded that for vertical flow the relative velocity between the air and the solids is equal to the single particle terminal velocity. Others have refuted this assumption and produced data to show that the relative velocity is not a constant but increases with increasing air velocity.

Basic analyses relating air/particle drag forces and "frictional" forces due to particle collisions have been published by Hinkle (1953) and by Wen & Galli (1971). In the Hinkle thesis the equation developed requires experimental data relating air and particle velocities before a solution for pressure drop can be found. The relationship which was found to correlate the measured particle and gas velocities is of an empirical nature and is of uncertain value outside the conditions in the test. Wen & Galli derive a general equation for horizontal flow which relates particle and gas velocities but its solution requires data on the solids frictions factor λ_s together with an iterative or graphical solution for the solid particle velocity in terms of the air velocity.

In the following analysis it is shown that in the case of vertical flow a general solution for particle velocities can be written in terms which include the solids friction factor λ_s and the multiparticle drag coefficient C_D. By considering only the excess pressure gradient due to the solids rather than the total pressure gradient it is shown that, providing $\lambda_s \neq 0$, the excess pressure gradient has a minimum value and that both λ_s and C_D can be determined by measuring the excess pressure gradient and the gas velocity co-ordinates at this minimum.

EQUATION OF MOTION OF AN ASSEMBLAGE OF PARTICLES

The equation of motion for a single particle which is transported upwards in an unbounded gas stream is similar to the well known case of a particle falling in free air. In fully established flow when the particle has no acceleration the drag force on the particle equals the gravitational force as follows,

$$C_{Dt}A_s\rho_G(u_G - u_s)^2/2 = mg, \qquad [1]$$

where A_s is the projected area of the particle, ρ_G is the gas density, u_G and u_s are the mean velocity of the gas and the solid, respectively and m is the mass of a single particle. In this case the relative velocity equals the single particle terminal velocity and,

$$u_G - u_s = u_t = (2mg/C_{Dt}A_s\rho_G)^{0\cdot 5}. \qquad [1a]$$

For spherical particles the mass m and the projected area of the particle A_s are given by,

$$m = \rho_s \pi d_s^3/6, \qquad [2]$$

and

$$A_s = \pi d_s^2/4, \qquad [3]$$

then [1a] can be expressed in the familiar form,

$$u_G - u_s = u_t = (4gd_s(\rho_s/\rho_G)/3C_{Dt})^{0\cdot 5}. \qquad [4]$$

The behaviour of multiparticle systems can be highly complex if the particles are small enough to be caught up in the small turbulent eddies in the fluid.

However the following discussion is restricted to a consideration of particles which are large compared with the scale of turbulence and in this case it is usual to consider the behaviour of the individual particles in the assemblage. We can, then, define a mean drag coefficient for the individual particles in the system which takes into account any changes in the velocity gradients due to the proximity of other particles as,

$$C_D = 2F_D/A_s\rho_G(u_G - u_s)^2 \qquad [5]$$

where F_D is the average drag force on one particle. The total drag force on N particles in a length ΔL of the pipe is then given by

$$\sum^N F_D = NC_DA_s\rho_G(u_G - u_s)^2/2. \qquad [5a]$$

Interparticle collisions and collisions with the pipe wall retard the particles. The situation is analogous to the energy loss due to pipe wall friction in single phase flow. Define a solids friction factor λ_s:

$$\lambda_s = \Delta P_f/2(\Delta L/D)\rho_{ds}u_s^2. \qquad [6]$$

then $\Delta P_f A$ is the retarding force on the particles due to collisions in the length ΔL.

The dispersed solids density, ρ_{ds}, is given by

$$\rho_{ds} = Nm/A\Delta L. \qquad [7]$$

and the retarding force due to collisions is then given by

$$\Delta P_f A = \lambda_s N m u_s^2/2D. \qquad [8]$$

Using [5a] and [8] and including a term, Nmg, to account for the gravitational force on the particles, we write an equation of motion in the form

$$Nm\ \mathrm{d}u_s/\mathrm{d}t = NC_D A_s \rho_G (u_G - u_s)^2/2 - Nmg - \lambda_s N m u_s^2/2D. \qquad [9]$$

For the case of fully established flow, $\mathrm{d}u_s/\mathrm{d}t = 0$, and it follows that,

$$C_D A_s \rho_G (u_G - u_s)^2/2m = \lambda_s u_s^2/2D + g. \qquad [10]$$

By combining [1a] and [10] we obtain

$$C_D/C_{Dt}((u_G - u_s)/u_t)^2 = \lambda_s u_s^2/2gD + 1. \qquad [11]$$

It should be noted that the use of [1a] rather than [4] makes [11] valid for particles of any shape. Duckworth (1974) has shown that a general equation for particle velocity can be obtained by re-arranging [11] in the form

$$Au_s^2 - 2Bu_s + C = 0 \qquad [12]$$

where

$$A = (1 - (\lambda_s u_t^2/2gD)(C_{Dt}/C_D)) \qquad [13]$$

$$B = u_G \qquad [14]$$

$$C = u_G^2(1 - (u_t/u_G)^2(C_{Dt}/C_D)) \qquad [15]$$

then

$$u_s = (B - (B^2 - AC)^{0\cdot 5})/A. \qquad [16]$$

It is interesting to note that the Duckworth equation [16] gives the satisfactory result that

$$u_G - u_s = u_t(C_{Dt}/C_D)^{0\cdot 5}, \qquad [17]$$

when $\lambda_s = 0$, i.e. when there are no collisions

and

$$u_G - u_s = u_t, \qquad [17a]$$

when $\lambda_s = 0$ and $C_D = C_{Dt}$ i.e. for a single particle in an unbounded air stream, [16] allows us to calculate u_s using known particle and system parameters providing λ_s and C_D are known.

Experimental data is required for the determination of C_D and λ_s but it is suggested that for lightly-loaded systems C_D will approximate to C_{Dt} and for relatively dense gas–solids mixtures the relationship,

$$C_D \approx C_{Dt} e^{-4.7} \qquad [18]$$

given by Wen & Galli (1971) may be used. In [18] the voidage is given by

$$e = 1 - w/\rho_s A u_s. \qquad [19]$$

The particle friction factor λ_s can be estimated from the data of Barth (1958, 1960) or Rose & Duckworth (1969).

DETERMINATION OF λ_s AND C_D FROM EXPERIMENTAL DATA

Even with solids velocity measurement it is difficult to isolate the independent variables λ_s and C_D over the range of air velocities encountered in gas–solids systems. However a simplified approach is possible which provides a discrete solution resulting from the following argument.

From a momentum analysis applied to the control volume containing a suspension of particles in air it can be shown that, for fully established flow, the following relationship holds,

$$(\Delta P_{\text{total}} - \Delta P_{\text{air}})/\Delta L = \Delta P/\Delta L = \rho_{ds} g + \Delta P_f/\Delta L, \qquad [20]$$

and

$$\rho_{ds} = Nm/A\,\Delta L = w/Au_s. \qquad [21]$$

Substituting for $\Delta P_f/\Delta L$ from [6] we obtain

$$\Delta P/\Delta L = w/A((g/u_s) + \lambda_s u_s/2D). \qquad [22]$$

If λ_s is constant then the minimum value of $\Delta P_f/\Delta L$ corresponds to the condition $d(\Delta P/\Delta L)/du_s = 0$ and hence differentiating [22] we obtain

$$u_{sm} = (2gD/\lambda_s)^{0.5}, \qquad [23]$$

and

$$(\Delta P/\Delta L)_{\min} = 2wg/A(\lambda_s/2gD)^{0.5}. \qquad [24]$$

Thus for a given solids mass flow rate w and pipe diameter D, λ_s can be found from measurement of the excess pressure gradient minimum.

By combining [23] and [24]:

$$u_{sm} = 2w_g/A(\Delta P/\Delta L)_{\min}. \qquad [25]$$

Also at the excess pressure gradient minimum, [11] can be written

$$C_D/C_{Dt}((u_{Gm} - u_{sm})/u_t)^2 = 2, \qquad [26]$$

where u_{sm} is given by [25] and C_D can be found from measurement of the gas velocity at the excess pressure gradient minimum. For lightly loaded systems $C_D \approx C_{Dt}$ and [26] becomes

$$u_{Gm} - u_{sm} \approx 1.41 u_t. \qquad [27]$$

EXPERIMENTAL VERIFICATION

Whilst it is recognised that a full experimental investigation is required to test the method over a range of pipe diameters and solids loadings, only limited test results obtained from a 50 mm pipe system at low solids flow rates are available.

The experimental results, plotted in figures 1 and 2, show the excess pressure gradient progressing towards an anticipated minimum as the air velocity is increased, but the curves are very flat and the minimum is not likely to be well defined in lightly loaded systems.

However, the flat curves make it easy to determine $(\Delta P/\Delta L)_{\min}$ and [24] has been used to

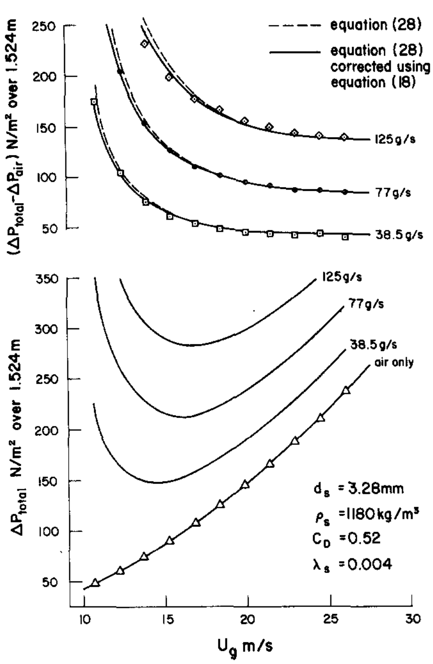

Figure 1. Acrylic spheres. Pressure drop over 1.524 m in a 47.6 mm pipe.

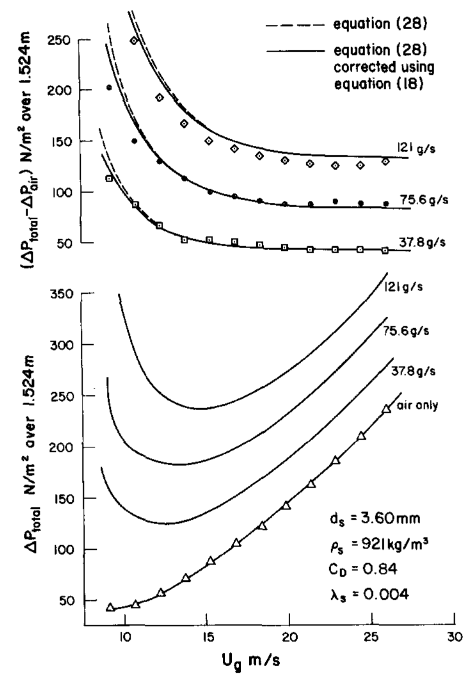

Figure 2. Alkathene chip. Pressure drop over 1.524 m in a 47.6 mm pipe.

determine values of λ_s from the experimental data for both acrylic spheres and polyethylene cylinders.

No variation between the two materials was found. In both cases $(\Delta P/\Delta L)_{min}$ was simply proportional to the solids flow rate w and a constant value of $\lambda_s = 0.004$ was obtained for both materials at the three different solids flow rates

Equation [16] requires a knowledge of C_D which according to [25] and [26] can be found from the co-ordinates of the excess pressure drop minimum. Since the minimum was not defined by the experimental results a computer program was used to find a value of C_D which best fitted the results. This value was corrected by using [18] to take account of the reduction in voidage at low velocities.

Equation [22] is plotted in figures 1 and 2 using a single value for C_D in each case and then again using [18] to correct C_D at low velocities. The variation of C_D with particle Reynolds number has been ignored for the small velocity range covered by the tests.

CONCLUSION

A general analytical solution for particle velocities can be written for the vertical flow of particulate solids in air. The solution for u_s is given by [16] in terms which include a solids friction factor λ_s and a multiparticle drag coefficient C_D. At the excess pressure gradient minimum the solution is independent of λ_s and [17] can be used to relate the solid and air velocities. If the multiparticle drag coefficient is approximately equal to the single particle drag coefficient then the slip velocity between the air and the particles will be greater than the single particle terminal velocity by a factor of 1.41 at the minimum for excess pressure gradient.

For lightly loaded systems operating in the limited velocity range normally encountered in particle conveying systems, good agreement between theory and experiment can be obtained by using [16] and [18] to calculate particle velocities and then [22] to calculate the excess pressure drop. Relatively simple experiments can be carried out to determine λ_s and C_D for a given pipe size. Particle velocity measurements may not be necessary but a range of pipe sizes should be investigated if the data is to be extrapolated to pipe sizes other than those used in any tests.

Experimental data for a range of pipe diameters and solids loadings is required to confirm the theoretical prediction of an excess pressure gradient minimum. The excess pressure gradient curve is very flat in the region of the minimum and high solid/air loadings are required to give a better definition of the minimum, particularly if the minimum value is used to calculate λ_s and C_D. The minimum can be expected to occur at the upper end of the range of velocities normally encountered in solids transport systems and at much higher velocities than the better known total pressure gradient minimum.

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REFERENCES

Barth, W. 1958 Strömungsvorgänge beim Transport von Festteilchen und Flüssigkeitsteilchen in Gasen. *Chemie-Ing Techn* **30**, 171–180.

Barth, W. 1960 Physikalische und wirtschaftliche Probleme des Transportes von Festteilchen in Flüssigkeiten und Gasen. *Chemie-Ing Techn* **32**, 164–171.

Boothroyd, G. R. 1971 *Flowing Gas–Solids Suspensions.* pp. 24–28. Chapman & Hall.

Duckworth, R. A. 1974 Private communication.

Duckworth, R. A. & Rose, H. E. 1969 Transport of solid particles in liquids and gases. *The Engineer* **227**, 5903, 392–396 (14 March 1969), 5904, 430–433 (21 March 1969) and 5905, 478–483 (28 March 1969).

Hinkle, B. L. 1953 Acceleration of particles and pressure drops encountered in horizontal pneumatic conveying. Ph.D Dissertation, Georgia Institute of Technology.

Kunii, D. & Levenspiel, O. 1969 *Fluidization Engineering*, 385–386. Wiley.

Wen, C. Y. & Galli, A. F. 1971 Dilute phase systems. *Fluidization* (Ed J. F. Davidson & D. Harrison), pp. 677–690. Academic Press.

Wen, C. Y. 1973 Estimating the solid particle velocity in vertical pneumatic conveying lines. *Ind Eng Chem Fundam.* **12**, 349–352.

Zenz, F. A. & Othmer, D. F. 1960 *Fluidization and Fluid Particle Systems*, pp. 477–484. Reinhold.